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Dust acoustic wave in a thermal dusty plasma

P. K. Shukla*
Institut für Theoretische Physik IV, Fakultät für Physik und Astronomie, Ruhr-Universität Bochum, D-44780 Bochum, Germany

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The dust acoustic wave (DAW) spectrum is reexamined to account for the dust charge fluctuation dynamics in a thermal dusty plasma whose constituents are electrons and positively charged dust particulates. The latter appear because of thermionic and ultraviolet induced photoemissions. Consideration of dust charge fluctuations modifies the DAW frequency, and also produces collisionless spatiotemporal damping.

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It is well recognized that dust acoustic (DA) [1] and dust ion-acoustic (DIA) [2] waves are the two normal modes of an unmagnetized dusty plasma, which is composed of electrons, ions, and micrometer sized extremely massive charged dust particulates. Both DA and DIA waves have been observed [3–7] in several low-temperature dusty plasma devices. However, in the DA and DIA fields, charges on the dust grain surface may fluctuate owing to the variation of the electron and ion currents that reach the dust grain surface during dust grain charging. The effects of dust charge fluctuations on DA and DIA waves were examined theoretically by Varma, Shukla, and Krishan [8], as well as by many other authors [8–11], and the charging equation was described by ''orbital-limited motion'' (OLM) electron and ion currents [12] for situations when the dust grains are negatively charged. The OLM theory is valid for $R \ll \lambda_D \ll \lambda_m$, where R is the radius of a spherical dust particle, λ_D is the effective dusty plasma Debye radius [13], and λ_m is the mean free path for electron-neutral and ion-neutral collisions. Recently, Morfill, Ivlev, and Jokipii [14] have demonstrated the amplification of a dust lattice wave due to stochastic dust charge fluctuations, while Kharpak *et al.* [15] examined the dynamical properties of random charge fluctuations in a dusty plasma with different charging mechanisms. The importance of dust charge fluctuations has also been recognized in some laboratory experiments [16,17]. Specifically, Chui and Goree [16] showed that very small grains can experience fluctuations to neutral and positive polarities, even in the absence of electron emission. On the other hand, Nunomura *et al.* [17] report instability of dust particles in a Coulomb crystal due to delayed charging associated with dust charge variations. A critical evaluation of the literature reveals that the previous investigations dealing with the effects of dust charge fluctuations on wave motions and instabilities have focused on a dusty plasma with negatively charged dust grains only.

However, in a thermal dusty plasma the dust grains are mostly charged positively [18] due to thermionic emission [19] and ultraviolet (UV) induced photoemission [20–22]. Hence, a thermal dusty plasma is composed of electrons and positively charged dust grains. There are no ions in the system. In this paper, we discuss the dust acoustic wave spectrum for this two-component dusty plasma and account for the dust charge fluctuation dynamics.

We consider the propagation of low frequency ($\omega \ll \nu_{eff}$, where ν_{eff} is the effective collision frequency defined in Ref. [23]), long wavelength [in comparison with $v_{te}/|\nu_{eff}(\nu_{ion}-\nu_{ed})|^{1/2} \equiv \lambda_{mf}$, where v_{te} is the electron thermal velocity and ν_{ion} and ν_{ed} are the ionization rate and the electron dust collision frequency, respectively] DA waves in an unmagnetized thermal dusty plasma whose constituents are electrons and extremely massive positively charged dust particulates. The dust sizes and the intergrain spacings are much smaller than the electron Debye radius. We also assume that there is a sufficient number of charged dust grains within the Debye sphere and the shielding of the dust grains comes from the background electrons. The electron number density in the electrostatic DA wave potential ϕ is [1]

$$n_e(\phi) = n_{e0}\exp(e\phi/T_e), \tag{1}$$

where n_{e0} is the unperturbed electron (ion) number density, e the magnitude of the electron charge, and T_e the electron temperature. The quasineutrality requires that $n_{e0}=Z_0 n_{d0}$, where Z_0 is the unperturbed number of positive charges residing on the dust grain surface and n_{d0} is the unperturbed dust number density. We stress that Eq. (1) is valid on a time (space) scale that is longer (smaller) than $\nu_{eff}(\lambda_{mf})$. Accordingly, the electron production and loss rates as well as the volume recombination rate are negligibly small.

The dust grain dynamics is governed by the continuity and momentum equations, which are, respectively,

$$\partial_t n_d + \nabla\cdot(n_d \mathbf{v}_d) = 0, \tag{2}$$

and

$$(\partial_t + \nu_d + \mathbf{v}_d\cdot\nabla)\mathbf{v}_d = -\frac{eZ_0}{m_d}\nabla\phi - \frac{3T_d n_d}{m_d n_{d0}^2}\nabla n_d, \tag{3}$$

where n_d and $\mathbf{v}_d$ are the dust number density and the dust fluid velocity, respectively, ν_d is the dust-neutral collision frequency, T_d is the dust temperature, and m_d is the dust mass.

Equations (1)–(3) are supplemented by the Poisson equation

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*Also at the Department of Plasma Physics, Umeå University, Umeå, Sweden.

$$\nabla^2\phi=4\pi e(n_e-Z_d n_d) \tag{4}$$

and the charging equation

$$(\partial_t+\mathbf{v}_d\cdot\nabla)Z_d=(I^+-I_e^-)/e. \tag{5}$$

The current I^+ is given by

$$I_{te}^+=4\pi R^2\left(\frac{m_eT_e}{2\pi\hbar^2}\right)^{3/2}\sqrt{\frac{2}{\pi}}en_{e0}v_{te}(1+\gamma)\times\exp(-\gamma-W_e/T_e) \tag{6}$$

for dust charging by thermionic emission [19], and

$$I_{pe}^+=\pi R^2eJY\exp(-\sigma\gamma) \tag{7}$$

for dust charging by UV irradiation [20]. The expression for the electron collection current, which is obtained from the OLM theory, is [12]

$$I_e^-=4\pi R^2en_e(\phi)v_{te}(1+\gamma), \tag{8}$$

where $\hbar$ is the Planck constant, $v_{te}=(T_e/m_e)^{1/2}$ is the electron thermal velocity, m_e is the electron mass, $\gamma=Ze^2/RT_e\equiv e\varphi/T_e$, φ is the surface potential of the dust grain, W_e is the work function, J is the UV photon flux, Y is the yield of photons, $\sigma=T_e/T_p$, and T_p is the average energy of the photoelectrons.

Letting $\varphi=\varphi_0+\varphi_1\equiv e(Z_0+Z_1)/R=eZ_d/R$, and assuming that $\varphi_1\ll\varphi_0$ and $e\phi/T_e\ll 1$, we obtain from Eqs. (1), (6), (7), and (8) for the fluctuating currents

$$\delta I_{te}^+=-Z_1\sqrt{\frac{2}{\pi}}\frac{R\omega_{pe}}{\lambda_{De}}\left(\frac{m_eT_e}{2\pi\hbar^2}\right)^{3/2}e\gamma_0\exp(-\gamma_0-W/T_e), \tag{9}$$

$$\delta I_{pe}^+=-Z_1\frac{R\omega_{pe}}{\lambda_{De}}\frac{eJY}{4\pi n_{e0}v_{te}}\sigma\exp(-\sigma\gamma_0), \tag{10}$$

and

$$\delta I^-=2\sqrt{2\pi}R^2en_{e0}v_{te}\left(\frac{Z_1e^2}{RT_e}+(1+\gamma_0)\frac{e\phi}{T_e}\right), \tag{11}$$

where $\omega_{pe}=(4\pi n_{e0}e^2/m_e)^{1/2}$ is the electron plasma frequency, $\lambda_{De}=(T_e/4\pi n_{e0}e^2)^{1/2}$ is the electron Debye length, and $\gamma_0=Z_0e^2/RT_e\equiv e\varphi_0/T_e$.

Substituting Eqs. (9)–(11) into Eq. (5), we readily obtain

$$(\partial_t+\nu_{1,2})Z_1=-f\omega_{pe}e\phi/T_e, \tag{12}$$

where

$$\nu_1=\frac{\omega_{pe}R}{\sqrt{2\pi}\lambda_{De}}\left[1+2\left(\frac{m_eT_e}{2\pi\hbar^2}\right)^{3/2}\right]\gamma_0\exp(-\gamma_0-W/T_e) \tag{13}$$

represents the frequency associated with dust charge perturbation during thermionic emission of electrons, and

$$\nu_2=\frac{\omega_{pe}R}{\sqrt{2\pi}\lambda_{De}}\left(1+\sqrt{2\pi}\frac{JY}{4\pi n_{e0}v_{te}}\sigma\exp(-\sigma\gamma_0)\right) \tag{14}$$

is the frequency associated with dust charge perturbation during the emission of electrons from the dust grain surface due to UV irradiation. Furthermore, we have denoted $f=4\pi n_{e0}\lambda_{De}R^2(1+\gamma_0)$.

On the other hand, letting $n_d=n_{d0}+n_{d1}$, where $n_{d1}\ll n_{d0}$, we obtain from Eqs. (2) and (3)

$$[(\partial_t+\nu_d)\partial_t-3v_{td}^2\nabla^2]n_{d1}=n_{d0}\frac{Z_0e}{m_d}\nabla^2\phi, \tag{15}$$

where $v_{td}=(T_d/m_d)^{1/2}$ is the dust thermal velocity. Combining Eqs. (1), (4), (12), and (15), and Fourier transforming the resulting equation ($\partial_t=-i\omega,\nabla=i\mathbf{k}$, where ω and $\mathbf{k}$ are the frequency and the wave vector, respectively), we obtain the dispersion relation

$$1+\frac{1}{k^2\lambda_{De}^2}-\frac{\omega_{pd}^2}{(\omega+i\nu_d)\omega-3k^2v_{td}^2}+\frac{F}{k^2\lambda_{De}^2}\frac{\omega_{pe}}{\nu_{1,2}-i\omega}=0, \tag{16}$$

where $F=4\pi n_{d0}\lambda_{De}R^2(1+\gamma_0)$.

Equation (16), which is the main result of our paper, can be analyzed numerically. However, in most plasmas, we have $\nu_d\ll|\omega|\ll\nu_{1,2}$. Accordingly, letting $\omega=\omega_r+i\omega_i$ in Eq. (16), where $\omega_i\ll\omega_r$, we obtain for $\omega\gg kv_{td}$

$$\omega_r\approx\frac{kC_D}{(1+k^2\lambda_{De}^2+F\omega_{pe}/\nu_{1,2})^{1/2}} \tag{17}$$

and

$$\omega_i\approx-\frac{F\omega_{pe}\omega_r^4}{2\nu_{1,2}^2k^2C_D^2}, \tag{18}$$

where $C_D=(Z_0T_e/m_d)^{1/2}$ is the DA velocity. Equations (17) and (18) are the frequency and the damping rate of the DA wave in a thermal dusty plasma containing electrons and positively charged dust grains.

The spatial damping rate for a real frequency can be deduced from Eq. (16) by letting $k=k_r+ik_i$. For $kv_{td}\ll\omega$ and $k\lambda_{De}\ll 1$, we obtain

$$k_r^2=k_i^2+\left(1+\frac{F\omega_{pe}\nu_{1,2}}{(\nu_{1,2}^2+\omega^2)}\right)\frac{\omega^2}{C_D^2} \tag{19}$$

and

$$k_i=\frac{F\omega_{pe}\omega^2}{2k_rC_D^2(\nu_{1,2}^2+\omega^2)}. \tag{20}$$

In summary, we have examined the DA wave spectrum in a thermal dusty plasma when the dust grains are charged positively. Accounting for dust charge fluctuations, which arise owing to fluctuating electron and thermionic/photoemission currents, we have derived a dispersion relation for the DA wave in which the restoring force comes from the pressure of the inertialess electrons and the dust mass provides the inertia. It is found that consideration of dust charge fluctuations modifies the DA wave frequency and its spatiotemporal damping rates. We emphasize that the phase velocity $[(Z_0T_e/m_d)^{1/2}]$ of the present DA wave in a thermal dusty plasma is much smaller than that of a DIA wave [2], as $Z_0/m_d \ll Z_i/m_i$, where $Z_i(m_i)$ is the ion charge (mass). The DA wave does not suffer the electron and dust Landau dampings as $v_{td} \ll \omega/k \ll v_{te}$. The results of our investigation are thus useful for understanding the propagation of nonthermal ultralow-frequency DA fluctuations that may arise due to a two-stream instability involving electron beams. The DA fluctuations may scatter off electromagnetic waves from the Earth's polar mesosphere which contains a distinct positive dust layer [24–26]. Furthermore, the DA wave in a thermal plasma can also be responsible for an attractive force [27] between positively charged dust grains, leading to Coulomb crystallization. The attractive force is attributed to an oscillating wake potential [27] $\phi_c \approx (2Q_t/\xi)\cos(\xi/L)$, where Q_t is the charge of a test particle, $\xi = |z - Ut|$, U is the test particulate velocity along the z axis, $L = \lambda_{De}[(U - V_0)^2 - C_D^2]^{1/2}/C_D$ is the effective attraction length, V_0 is the z component of the dust flow velocity, and $|U - V_0| \ll C_D$. The wake potential is attractive for $\cos(\xi/L) < 0$. The physics of positive dust grain attraction is similar to Cooper pairing in which positively charged dust grains polarize the medium by attracting electrons when $\omega = k_zV_0 \pm k_zC_D$. The excess electrons attract the neighboring positive dust grains in the negative wake potential that is associated with the DAW in a thermal dusty plasma. Thus, quasilattice structures composed of positive dust grains are possible, as has been demonstrated in experimentally [18].

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